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A general number-to-space mapping deficit in developmental dyscalculia

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ABSTRACT

Previous research on developmental dyscalculia (DD) suggested that deficits in the number line estimation task are related to a failure to represent number magnitude linearly. This conclusion was derived from the observation of logarithmically shaped estimation patterns. However, recent research questioned this idea of an isomorphic relationship between estimation patterns and number magnitude representation. In the present study, we evaluated an alternative hypothesis: impairments in the number line estimation task are due to a general deficit in mapping numbers onto space.

Adults with DD and a matched control group had to learn linear and non-linear layouts of the number line via feedback. Afterwards, we assessed their performance how well they learnt the new number-space mappings. We found irrespective of the layouts worse performance of adults with DD. Additionally, in case of the linear layout, we observed that their performance did not differ from controls near reference points, but that differences between groups increased as the distance to reference point increased.

We conclude that worse performance of adults with DD in the number line task might be due a deficit in mapping numbers onto space which can be partly overcome relying on reference points.

1. Introduction

Developmental dyscalculia (DD) describes a heterogeneous disorder of numerical or arithmetic abilities in children (Kaufmann et al., 2013) which persist into late adolescence (Shalev, Manor, & Gross-Tsur, 2005) and even adulthood when untreated (e.g., Ashkenazi, Rubinsten, & Henik, 2009; Defever, Göbel, Ghesquiere, & Reynvoet, 2014; Mejias, Grégoire, & Noël, 2012; Rubinsten & Henik, 2005). Research on mathematical difficulties is of major importance, because mathematical difficulties have been shown to be more detrimental to career prospects than reading deficiencies (Parsons & Bynner, 2005). Additionally, Beddington et al. (2008; see also Goswami, 2008) argued that untreated mathematics learning difficulties can lead to immense societal costs.

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Deficits associated with DD have been argued to be driven by either domain-general (e.g., working memory; e.g., Andersson & Lyxell, 2007; Bartelet, Ansari, Vaessen, & Blomert, 2014; Geary, 2004; Kaufmann & von Aster, 2012) or domain-specific impairments (i.e., number representations or arithmetic skills; Kaufmann et al., 2013). Among domain-specific deficits, deficits in the representation of number magnitude were suggested as a core deficit of DD (Butterworth, 2005; Wilson & Dehaene, 2007), because it is the central semantic information conveyed by numbers and relevant in the majority of numerical tasks. Two main hypotheses have been put forward to account for deficits in the processing of number magnitude in DD: (1) a specific deficit in the magnitude system (Landerl, Bevan, & Butterworth, 2004; Wilson & Dehaene, 2007) and (2) a deficit in the accessing a magnitude representation from symbolic numbers (e.g., De Smedt, Noël, Gilmore, & Ansari, 2013; Rousselle & Noel, 2007). According to the first hypothesis, the mental representation of number magnitude is impaired which in turn causes numerical deficits in DD. In contrast, the access deficit hypothesis claims that the magnitude representation itself is intact, but access to this representation and thus the connection between the magnitude representation and symbolic numbers is deficient.

Several tasks are employed to measure deficits in the magnitude representation of numbers. Children and adults with DD were found to perform worse in, amongst others, symbolic and non-symbolic number comparison, automatic and intentional number magnitude processing as in the number Stroop task (e.g., Ashkenazi et al., 2009; Rubinsten & Henik, 2005) as well as number line estimation (e.g., Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Landerl, 2013; Landerl & Kolle, 2009).

Number line estimation, in particular, seemed to be a reliable indicator for identifying deficits of the mental representation of number magnitude, as it was suggested to provide a direct measure of the magnitude representation of symbolic numbers (Siegler & Booth, 2005; Siegler & Opfer, 2003). Confirming this suggestion, number line estimation patterns of children with DD were less accurate compared to typical achieving children (Geary et al., 2007; Geary et al., 2008; Landerl, 2013; Landerl, Fussenegger, Moll, & Willburger, 2009).

Recent research questioned the direct relationship between number line estimation performance and magnitude representation (Barth & Paladino, 2011; Huber, Moeller, & Nuerk, 2014; Karolis, Iuculano, & Butterworth, 2011; Moeller & Nuerk, 2011). Barth and Paladino (2011) pointed out that proportion-judgment strategies are applied when solving the number line estimation task (Hollands & Dyre, 2000). Proportion-judgment strategies imply that participants use reference points when estimating the correct position of a given number which results in a characteristic pattern of over- (for numbers smaller than the reference point) and underestimation (for numbers larger than the reference point) close to the respective reference points. Therefore, when children use the start and end point as references, an inverse S-shaped estimation pattern is to be expected which can be fitted using cyclic power models. Barth and Paladino (2011; see also Slusser, Santiago, & Barth, 2013) showed that cyclic power models provided superior fits to children's estimation patterns. From this the authors concluded that children indeed rely on reference points when estimating the position of numbers on a number line.

Consequently, performances differences in the number line estimation task between typically achieving children and children with DD might be attributable to both a deficit in mapping numbers onto physical space and a deficit in applying proportional judgment strategies. Using the standard linear number line estimation task it is not possible to differentiate between these two alternatives, because either deficit leads to poorer performance. For instance, longitudinal data of Landerl (2013) revealed that the performance of typically achieving children as well as children with DD improved from the beginning of grade 2 to the beginning of grade 4. This finding might be interpreted in line with the proportional judgment account: the number of children applying proportion judgment strategies increases with age which in turn improves their precision in the number line estimation task (Slusser et al., 2013). Nevertheless, although performance differences decreased from the beginning of grade 2 to the beginning of grade 4, typically achieving children outperformed children with DD in the number line estimation task on all grade levels. The initial (at grade 2) as well as the residual (at grade 4) performance difference can be either attributed to a deficit in mapping numbers onto physical space or a deficit in applying proportional judgment strategies in those with DD. On the one hand, children with DD might be able to compensate only partially for a mapping deficit by applying proportional judgment strategies. Hence, the residual performance difference might reflect a mapping deficit. On the other hand, children with DD might have a particular problem in applying the proportion judgment strategy, e.g., choosing good reference points.

Therefore, in present study, we instead used a newly developed number line learning task to investigated number-space mapping deficits which has already been successfully used in healthy adults and typically developing children (Huber et al., 2014). In this task, participants are trained to acquire different number line layouts (i.e., linear, logarithmic, exponential, sigmoid and inverse sigmoid) by giving them feedback on their initial estimates during a training phase. Healthy adults have no problem learning any linear and non-linear layout quite well in less than 3 min and even young children were able to learn different layouts to a certain extent (Huber et al., 2014).

Importantly, the task allows for differentiating between the two hypotheses of (1) a deficit in mapping numbers onto physical space and (2) a deficit in applying proportion judgment strategies. According to hypothesis (1), adults with DD relying heavily on reference points when solving a linear number line estimation task should have particular problems learning non-linear number line layouts. This is due to that nobody usually knows where 50 is located in an unknown logarithmic or exponential function. Therefore, number-space deficits in non-linear number line layouts cannot be easily compensated for by relying on reference points. A deficit in mapping numbers onto physical space should be reflected in a worse performance of adults with DD in non-linear layouts supporting the hypothesis of a general mapping deficit in the number line estimation task. In contrast, according to hypothesis (2), adults with DD experience should not experience particular problems in mapping numbers onto physical space resulting in a similar performance of adults with DD and the control group in non-linear layouts. This would suggest that performance difference in linear number line estimation might be due to a deficit in successful application of proportion judgment strategies.

Additionally, we examined whether a deficit in mapping numbers onto physical space might also be prevalent in the linear layout. We expected both adults with and without DD to use reference points which should be reflected in similar estimation errors around reference points for both groups. However, in accordance with the mapping deficit hypothesis (1), we expected larger estimation errors of adults with DD for numbers farther away from reference points.

2. Method

2.1. Participants

Thirty-six native Hebrew speaking adults participated in the study. Twenty-one healthy adults (see Table 1) were recruited through advertisements that were distributed on the Haifa University campus. Additionally, fifteen adults who had been diagnosed with DD (see Table 1) were recruited through a search in the diagnoses database of the clinic for learning disabilities of Haifa University (students diagnosed in the clinic are typically asked to sign a waiver that allows their test scores to be used for research purposes) and advertisements distributed on the university campus as well as at nearby colleges. Participants gave written consent to participate in the experiment and were paid 30 Shekels for their participation.

2.2. Ethics statement

The recruitment, payment, tasks and overall procedure were authorized by the Research Ethics Committee of Haifa University.

2.3. Classification and assessment criteria

All participants were classified as control or DD, using the ‘‘Israeli learning function diagnosis system’’ (titled in Hebrew also as ‘‘MATAL’’) for high school and higher education students developed by the National Institute for Testing & Evaluation (for more details, see e.g., Kennet-Cohen, Bronner, & Intrator, 2008). This diagnostic tool is composed of a set of standardized computerized tests and questionnaires intended for diagnosing learning disabilities in high school and higher education students (see Table 1). All tests and questionnaires are nationally normalized.

All participants performed standardized numerical (simple calculation, procedural knowledge and numbers line estimation) tasks, reading and rapid naming tasks, and an attention task (continuous performance test – CPT). Participants also answered a questionnaire (based on DSM) regarding their childhood and adulthood attention ability (see Table 2).

The cut-off inclusion criterion was a score below (for the DD group) or above (for the control group) the 20th percentile in either RT or accuracy (ACC) on the simple calculation and the procedural knowledge subtests, and a score above the 10th percentile (for both groups) in the reading and attention subtests (see Table 2). The criteria for the DD vs. control groups (for all screening tests) was chosen after consulting with (a) several clinicians who use the MATAL diagnosing tool (at the

Table 1
Description of the assessment tests from the MATAL used in the current study (Kennet-Cohen et al., 2008).

Test	Skill/function	Task description	Performance measures
Mathematics			
Simple calculation	Retrieval of simple arithmetical facts	Judging the correctness of simple arithmetic equations (e.g., judging whether the equation 5 + 4 = 9 is correct or wrong)	• Accuracy • RT
Procedural knowledge	Mastery of basic arithmetic problems	Judging the correctness of arithmetic equations (e.g., judging whether the equation 156 – 67 = 89 is correct or wrong)	• Accuracy • RT
Numbers line estimation	Number line representation	Determining which of two numbers values presented on a number-line corresponds to a given number	• Accuracy • RT • Distance related accuracy
Reading			
Text reading – ACC	Phonological decoding	Reading of a non-vocalized text	• Accuracy
Rapid automatic maiming (RAN)	Lexical retrieval	Rapid naming of objects, letters and numbers	• Naming rate
Attention			
Attention (CPT)	Sustained attention	Response to a specific two dimensional target stimuli (shape and color) that appears randomly within a range of other stimuli	• Omissions • Commissions in 1st part • Commissions in 2nd part • RT • Variability of RT

Table 2
Descriptive information and mean percentile scores in the selection tasks for DD and control groups (ACC = accuracy, RT = reaction time; m = months, y = years).

	Control group	DD group	*t*
Descriptive information			
N	21	15	
Gender (M/F)	4/17	1/15	
Age	25y, 8 m (*SD* = 3y, 1 m)	27y, 3 m (*SD* = 4y, 2 m)	
Mathematics			
Simple calculation – ACC	51–78	6–10	5.50***
Simple calculation – RT	50–60	8–10	4.52**
Procedural knowledge – ACC	54–59	4–5	1.45
Procedural knowledge – RT	57–59	10–13	5.30**
Numbers line estimation – ACC	41–45	10–11	4.10*
Distance relates accuracy	51–70	22–35	1.45
Numbers line estimation – RT	41	53	−3.45*
Reading			
Text reading – ACC	57	58–78	0.54
Rapid naming – letters	84–88	67–71	2.61*
Rapid naming – numbers	70–74	35–44	2.51*
Attention (CPT)			
Omissions	20–38	20–38	−0.55
Commissions 1	33–67	17–33	1.27
Commissions 2	52–81	52–81	1.30
RT	38	52	−0.71
Variability of RT	39	55	−0.44

Note: Standard deviations are shown in parentheses.
* $p < .05$.
** $p < .01$.
*** $p < .001$.

University of Haifa diagnosing center for student with learning disabilities), (b) relying on the MATAL guideline and manual, and (c) reviewing several papers investigating participants with Dyscalculia or MLD. We found the criteria for dyscalculia to vary widely across papers from the 10th percentile (e.g., Shalev, Auerbach, Manor, & Gross-Tsur, 2000) up to the 35th percentile in some cases (25th percentile: Hanich, Jordan, Kaplan, & Dick, 2001; 35th percentile: Jordan, Hanich, & Kaplan, 2003; 30th percentile: Geary, Hoard, Byrd-Craven, & Catherine DeSoto, 2004; 15th percentile: Rousselle & Noël, 2007; 15th percentile: Mussolin & Noel, 2008). Hence, we chose a criterion that followed the recommendation of the clinicians and MATAL guidelines and falls in the lower more conservative end of the criteria applied in previous papers.

By applying these criteria, within the DD group, each participant scored under the 10th percentile in either RT or accuracy in both the simple calculation and procedural numerical knowledge task. Hence, the DD group actually demonstrates severe difficulties in calculation tasks conforming to the strictest criteria used in the literature.

Additionally, concerning the criteria for having a reading difficulty, it is important to mention that all reading scores of individuals in both the DD and the control group are above the 15th percentile and at least three out of the four reading related scores are above the 20th percentile.

Note that mean scores for RT in the number line estimation task suggest that DD participant were faster than controls in this task (although significantly less accurate). Therefore, it is important to note that the number line estimation task is a complex task that requires the subject to (1) turn the participant's attention to the range of the number line (which changes from trial to trial), (2) to ignore distracter markers on the number line, and (3) to attend to the direction of the keys with regard to the options that appear on the screen (see Fig. 1). It is possible that this task was difficult for the DD group and hence was performed quickly – this may explain the faster RT scores. Note that the instruction focused on accuracy and not on RT.

2.4. Stimuli and design

In the experimental task participants were requested to mark the position of a given number in a computerized number line estimation task (range 0–100). The task comprised five different number line layouts (i.e., linear, logarithmic, exponential, sigmoid and inverse sigmoid; see Huber et al., 2014 and see also Fig. 2). In a first training phase participants had to learn the respective number line layout through the feedback which was provided after each trial. The training phase included the following 30 target numbers: 1, 2, 8, 13, 15, 17, 26, 27, 29, 31, 38, 39, 42, 43, 49, 52, 53, 54, 63, 65, 67, 71, 73, 78, 82, 86, 87, 94, 97, and 98. In the subsequent testing phase participants had to locate the position of the following 20 numbers without receiving feedback anymore: 3, 9, 12, 16, 23, 25, 34, 36, 41, 47, 51, 56, 61, 68, 74, 79, 85, 89, 95, and 96. Numbers were presented in a randomized order. Additionally, we varied the order of the number line layouts following a Latin square design.

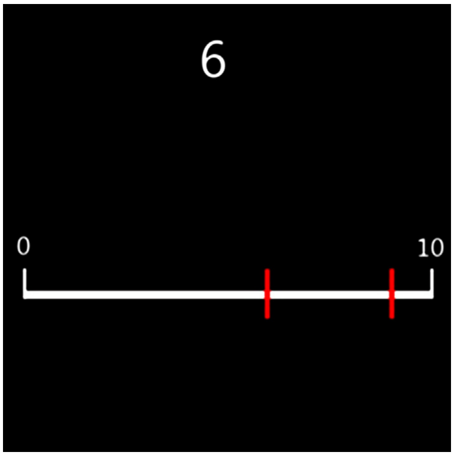

Fig. 1. An illustration of the number line task in the "MATAL": participants are asked to decide which of the two red markers is compatible to the number presented above the number line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The task was programmed in Java 6.0 using Java Swing and presented on a standard PC with a 17″ monitor. Screen resolution was set to 1024 × 768 pixels. The number line started at the coordinates 170/460 and ended at the coordinates 853/460. Thus, it was 683 pixels long (about 18.5 cm). Centrally above the line the respective target numbers were presented (511/225). We used Sans Serif as font for the numbers (size: 60, style: bold).

2.5. Procedure

The experimental task and the other assessment tasks were performed on different days with assessment tasks always performed prior to the experimental tasks.

Participants were seated about 60 cm from the monitor. Testing took place in a quiet room at Haifa University. The instruction at the start of the experiment informed participants that in the following task they had to locate given numbers on a number line for which they should learn the correct number-to-space mapping by trial and error during the training phases. Estimations were made by clicking on the number line with the mouse cursor which was changed to a blue dash. In the training phase, participants received feedback by a green dash located at the correct position which was given immediately after the response and lasted for 2 s. At the end of the training phase, participants were given feedback about their performance and they instructed that in the following testing phase there would be no feedback. After they had completed the testing phase, feedback about their performance was again presented before they were informed that the underlying function would change for the next run.

2.6. Analysis

Functions were fitted using the curve fitting toolbox of Matlab R2013b (see the Appendix for functions used in the fitting process). Statistical analyses were run using R (R Development Core Team, 2015), and the R package lme4 for linear mixed model analyses (Bates, Maechler, Bolker, & Walker, 2014). We analyzed reaction times in the training phase as a measure of training durations and absolute estimation errors in the test phase. Reaction times were calculated as the time required locating the mouse cursor on the number line per trial and absolute estimation error as the absolute difference between correct position and estimated position on the number. In the analyses of reaction times and absolute estimation errors, we considered group (DD vs. control) and function (linear, logarithmic, exponential, sigmoid and inverse sigmoid) as factors. Therefore, we entered these predictors as well as their interaction as fixed effects into the linear mixed models (LMM). Furthermore, we used the maximal random effects structure including random intercepts for participants and items and a random slope for function as recommended by Barr, Levy, Scheepers, & Tily (2013). By considering participants and items as random intercepts, we assumed that mean RT and absolute estimation errors varied between participants and items. Moreover, the random slope for function indicated that we assumed that the effect of function differed between participants. Using the maximal random effects structure ensures nominal Type I error rates. Predictors were effect coded prior to data analyses to get Type III tests of fixed effects (default for ANOVAs in SPSS and SAS). Using Type III tests, each effect is adjusted for all other effects and interactions in the model when testing the significance of the particular effect (e.g., see Langsrud, 2003). There are several possibilities to obtain *p*-values for estimated parameters in LMM. We used the Satterthwaite approximation for degrees of freedom available in the R package lmerTest (Kuznetsova, Brockhoff, & Christensen, 2014), because it provides finite-size corrections and results can be presented in an ANOVA-like output format (for other methods to obtain *p*-values for LMM, see Bates et al., 2014).

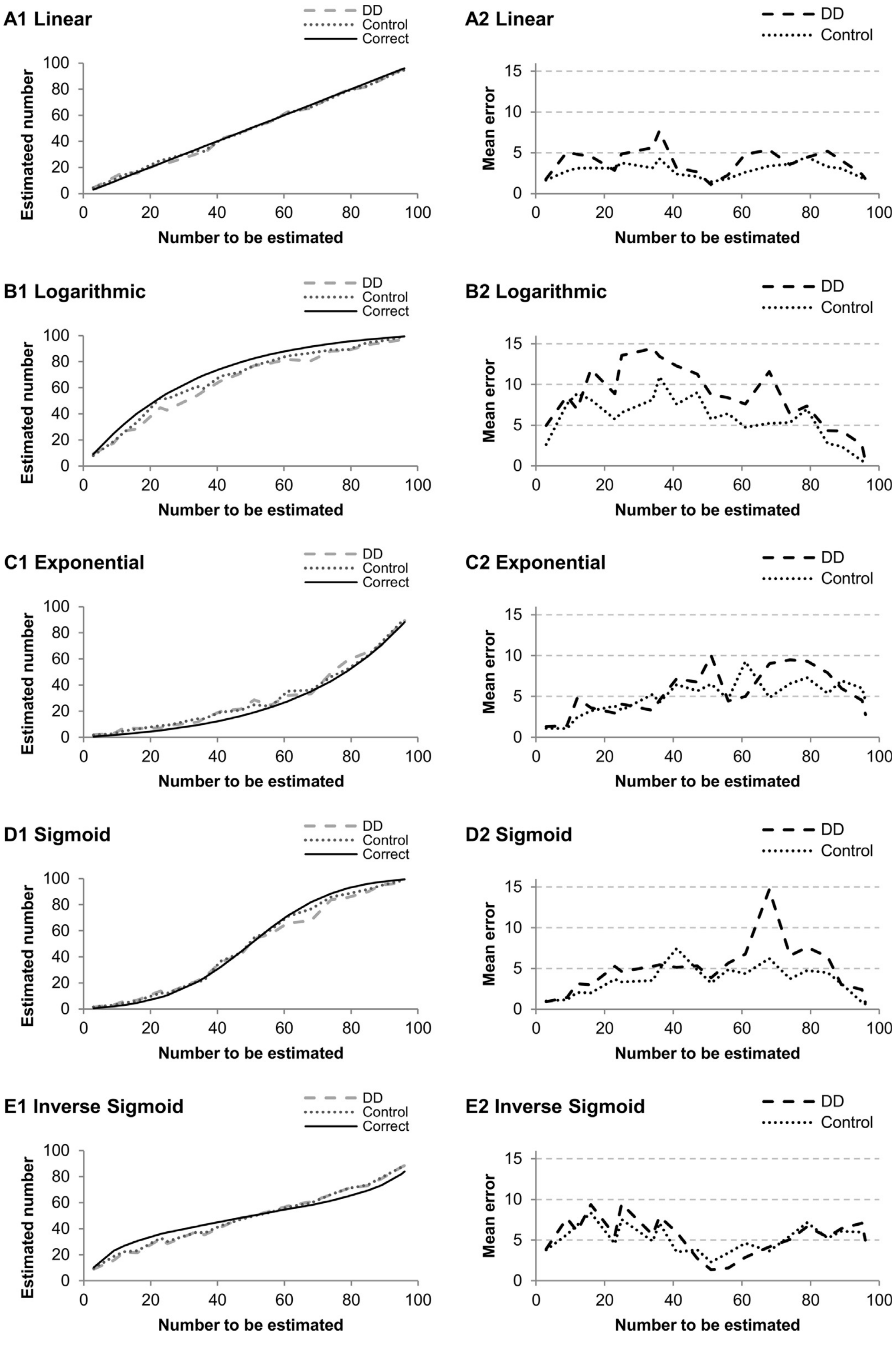

Fig. 2. Estimates (A1–E1) and absolute mean estimation error (A2–E2) of adults with dyscalculia (DD) and the control group (control) separately for each function (linear, logarithmic, exponential, sigmoid, and inverse sigmoid). Mean absolute estimation errors (mean error) were higher for the DD group for most estimated numbers.

3. Results

In a first step, we tested whether reaction times in the training phase as a measure of training durations differed between functions and groups. Main effects of group, function as well as the interaction between group and function were not significant indicating that reaction times did not differ between participant groups and number line layouts [all $F < 1.22$, all $p > .32$]. Next, we checked whether participants learnt the respective number line layouts through the training by fitting participants' estimates separately for each number line layout. The control group as well as the DD group learnt all functions quite well with adjusted $R^2 > 0.94$. Means of adjusted R^2 of the control group for the linear, logarithmic, exponential, sigmoid and inverse sigmoid function were, 0.99, 0.96, 0.96, 0.99, and 0.95, respectively and means of adjusted R^2 of the DD group were 0.97, 0.94, 0.95, 0.97, and 0.95, respectively (see also Fig. 2A1–E1).

Next, we evaluated whether groups differed in their absolute estimation error after training. LMM revealed significant main effects of group [$F(1, 35.95) = 4.16$, $p = .049$] and function [$F(4, 35.99) = 9.58$, $p < .001$], but no significant interaction [$F(4, 35.99) = 1.36$, $p = .268$]. The main effect of group indicated that estimates of the control group were reliably closer to the correct positions than the estimates of the DD group ($M = 4.47$, $SE = 0.45$ vs. $M = 5.62$, $SE = 0.50$). Additionally, mean absolute estimation error differed between the respective number line layouts. Mean absolute estimation errors differed significantly at least at .05 between number line layouts except for the exponential and inverse sigmoid layout (see Table 3). The non-significant interaction of the two factors indicated that group differences did not differ reliably between number line layouts. In other words, the DD group performed worse than the control group irrespective of number line layout including the linear as well as non-linear layouts. In Fig. 2, mean absolute estimation errors are depicted separated for the two groups.

In addition to the overall worse performance of the DD group, the plots for mean absolute estimation error of the linear function (Fig. 2A2) suggested that the DD group performed similar to the control group around the reference points 0, 25, 50, 75 and 100, but worse for numbers more distant from these reference points. Thus, the DD group seemed to rely on a proportion judgment strategy as did the control group, but participants with DD had particular problems estimating target numbers further away from these reference points. To test this hypothesis, we calculated the distance to reference points for each item. More precisely, for each target number we determined the closest reference point and computed the absolute difference between target number and reference point. For instance, the closest reference point for target number 17 was 25. Accordingly, the distance to the reference point for target number 17 was 8 (i.e., 25–17). Distances of target numbers 3, 9, 12, 16, 23, 25, 34, 36, 41, 47, 51, 56, 61, 68, 74, 79, 85, 89, 95, and 96 thus were 3, 9, 12 (to reference point 0), 9, 2, 0, 9, 11 (to 25), 9, 3, 1, 6, 11 (to 50), 7, 1, 4, 10 (to 75), 11, 5, and 4 (to 100), respectively.

Then, we ran a LMM with distance to reference point, group and its interaction as fixed effects and intercept of participants and items as random effects. Thus, our analysis included a continuous predictor (i.e., distance to reference point) and a categorical predictor (i.e., group) as well as their interaction. Moreover, we did not center the predictor distance to reference point. Therefore, the main effect of group has to be evaluated with respect to the continuous predictor distance to reference point which was set to zero at the reference points. Consequently, a non-significant main effect of group would indicate that at reference points (i.e., where distance to reference point = 0) estimates of groups would not differ. However, an interaction between group and distance to reference point would suggest that the difference between groups depends on the distance to reference point. We did not include distance to reference points as random slope, because the model including it as a random slope did not converge.

The analysis confirmed our assumption as the main effect of group was not significant [$F(1, 120.47) = 0.29$, $p = .594$], but the main effect of distance to reference points [$F(1, 20.15) = 6.72$, $p = .017$] as well as the interaction of the two factors were significant [$F(1, 664.62) = 12.89$, $p < .001$]. The absence of a main effect of group indicated that at reference points mean absolute estimation error did not differ significantly between groups ($M = 2.29$, $SE = 0.55$ vs. $M = 2.58$, $SE = 0.51$). The main effect of distance to reference points suggested that on average across groups, participants' estimation error increased as the distance from the reference points increased for the target numbers (estimated slope = 0.15, $SE = 0.06$). Importantly, this main effect was qualified by group as indicated by the significant interaction which indicated that differences between groups increased as the distance to reference point increased [control group: estimated slope = 0.04, $SE = 0.06$, $z = 0.69$, $p = .699$; DD group: estimated slope = 0.25, $SE = 0.07$, $z = 3.82$, $p < .001$; see also Fig. 3]. At the largest distance to reference points (i.e., 12), the estimated difference between groups was 2.24 ($SE = 0.52$; $z = 4.31$, $p < .001$).

Table 3
Estimates and standard errors of absolute estimation errors for each function and differences between absolute estimation errors of functions.

Condition	Estimate	SE	1	2	3	4
1. Linear	3.38	0.33	–			
2. Logarithmic	7.10	0.76	-3.7^{***}	–		
3. Exponential	5.17	0.44	-1.8^{***}	1.9^{**}	–	
4. Sigmoid	4.19	0.36	-0.8^{**}	2.9^{***}	1.0^{*}	–
5. Inverse sigmoid	5.39	0.49	-2.0^{***}	-1.7^{**}	−0.2	-1.2^{*}

* $p < .05$.
** $p < .01$.
*** $p < .001$.

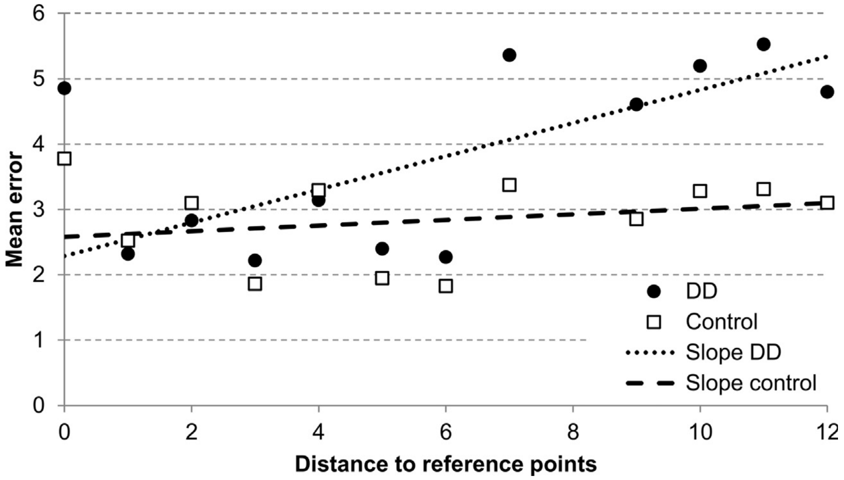

Fig. 3. Interaction between group (adults with and without dyscalculia) and distance to reference point. DD = adults with dyscalculia, control = control group, and mean error = mean absolute estimation error. Slope DD = estimated linear slope for adults with dyscalculia, slope control = estimated linear slope for the control group.

4. Discussion

In the present study, we aimed at investigating what is causing the worse performance of adults with DD in the number line estimation task. First, we tested whether they have a general problem in mapping numbers onto space by contrasting the performance in learning new number line layouts for a group of adults with DD against a control group. We observed that irrespective of the number line layout, estimates of the DD group were less precise than that of the control group. This indicates that adults with DD experienced general problems in learning new number line layouts and, hence, mapping numbers onto space. In particular, the worse performance of adults with DD for the non-linear number line layouts supports the idea of a general mapping deficit. There were no easy to identify reference points in the non-linear number line layouts except the start and end points of the number line. Therefore, adults with DD cannot rely on reference points and, thus, proportion judgment strategies which means that they had to estimate the location of numbers directly resulting in a significant drop of performance.

Second, we examined whether adults with DD try to compensate for the general number to space mapping deficit by making use of reference points, when this is possible (i.e., in the linear layout). Indeed, estimates of adults with DD at and around reference points did not differ from the estimates of the control group. However, their estimates were reliably less precise for target numbers further away from reference points. This finding suggests that adults with DD may try to overcome their deficit in estimating the correct location of numbers on a number line by relying on proportion judgment strategies making use of reference points.

The two main hypotheses accounting for the deficient processing of numerical magnitude in those with DD are the magnitude system deficit hypothesis (Landerl et al., 2004; Wilson & Dehaene, 2007) and the access deficit hypothesis (e.g., De Smedt et al., 2013; Rousselle & Noel, 2007). As in our task participants had to learn new number line layouts and, thus, had to flexibly map number symbols onto space, our results seem to be more in line with the idea of an access deficit hypothesis (e.g., De Smedt et al., 2013; Rousselle & Noel, 2007). The access deficit hypothesis assumes that those with DD suffer from a deficit in linking number symbols with the representation of the (numerical) magnitude conveyed by these symbols. This finding is in partial contrast to previous studies examining deficits of adults with DD. Mejias et al. (2012) found a deficit in symbolic as well as non-symbolic estimation tasks and, therefore, suggested that adults with DD have less precise magnitude representations per se. In contrast, Defever et al. (2014) examined deficits of adults with DD in symbolic and non-symbolic priming tasks. They observed similar priming distance effects for adults with and without DD indicating no significant difference in the underlying representation of number magnitude. Furthermore, these authors found no evidence for the access deficit hypothesis, as RT of adults with and without DD did not differ in the symbolic priming task.

However, several recent studies revealed that performance in the number line estimation task does not necessarily measure the representation of symbolic numbers (e.g., Barth & Paladino, 2011; Huber et al., 2014; Karolis et al., 2011). Thus, the present finding of a general mapping deficit should not be interpreted as evidence that adults with DD have a deficit in linking symbolic numbers with mental magnitude representations in the sense of the process that associates semantic meaning to the number symbols (see De Smedt et al., 2013; Feigenson, Libertus, & Halberda, 2013, for recent reviews on this issue). Instead, our results suggest that adults with DD have a particular deficit in mapping symbolic numbers onto physical space.

This process requires visual-spatial resources (Gunderson, Ramirez, Beilock, & Levine, 2012; LeFevre et al., 2013) and at least, in case of our experiment also good visuo-spatial memory, as participants had to learn new mappings. Thus, the deficit of adults with DD might also be due to a deficit in visual-spatial memory. Interestingly, a recent study by Szucs, Devine, Soltesz, Nobes, & Gabriel (2013) disconfirmed the magnitude system deficit hypothesis, but showed that visual-spatial memory and inhibitory control were specifically impaired in individuals with DD. Our present findings are in line with the interpretation that adults with DD have a particular deficit in visual-spatial memory. Moreover, our results indicate that adults with DD seem to try to overcome this deficit by relying on references points, but fail to do so when estimating numbers farther away from reference points. This indicates that the impairment is specific to the mapping of symbolic numbers onto a visual line.

On the other hand, the representation of space itself seems to be unimpaired, as adults with DD were able to divide the physical number line in equal partitions, when they were applying the reference point strategy to overcome their deficit in estimating the correct position of numbers. This was quite successful for the linear number line layout but did hardly work out for the non-linear layouts. An unimpaired representation of space (or visuospatial mechanisms) is in line with the results of Mussolin, Martin, & Schiltz (2011) who also found no differences between adults with DD and unaffected controls in judging whether the number is spatially located in between to other numbers. However, adults with DD were slower when they had to evaluate whether the number presented in the middle of two numbers reflects the numerical middle between the surrounding numbers.

Thus, the present findings suggest that visual-spatial memory, but not visual abilities per se are deficient in adults with DD. Good visual-spatial memory is not only required in the number line estimation task, but the visual-spatial sketchpad (i.e., visual-spatial working memory) was found predictive of mathematics achievement (Geary, 2011) Additionally, visual-spatial memory and general spatial orientation skills play an important role in mathematical development (Dumontheil & Klingberg, 2012; Geary et al., 2007; Geary et al., 2008; Metcalfe, Ashkenazi, Rosenberg-Lee, & Menon, 2013; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2014). This relationship offers an explanation, why children with DD not only perform worse in the number line estimation task (Landerl, 2013), but also struggle more generally with mathematical tasks.

Our findings are also important for assessing developmental dyscalculia using the number line estimation task. We observed that the performance of adults with DD and controls was similar around reference points in case of the linear layout. Hence, performance differences between adults with and without DD were largest for target number off the reference points. This is of particular interest as it indicates that specific difficulties in number line estimation associated with DD may be detected best for those target numbers. However, our experiment also showed that it is also possible to bypass this issue by employing a non-linear number line estimation task for which it is unlikely that participants may know the location of reference points. Although we found no reliable interaction between participant group and number line layout the performance discrepancy between adults with DD and the control group were descriptively largest in case of the logarithmic layout. Therefore, the present results indicate that non-linear versions of the number line estimation task, and in particular the logarithmic layout might be best suited for differentiating between adults with and without DD by means of a number line estimation task.

Furthermore, the present results also have implications for DD intervention and remediation. First, the present study suggests that individuals with DD use particular strategies to overcome their deficit in mapping number onto space. Thus, explicitly teaching such strategies might be a promising way to reduce specific outcome deficits of those with DD. On the other hand, this may not address the underlying difficulty. Importantly, however, we observed a particular deficit in visual-spatial working memory for those with DD. In this context, a recent meta-analysis by Melby-Lervåg and Hulme (2013) revealed that working memory training did not improve arithmetic performance considerably (i.e., they found a non-significant small effect size). In contrast, two more recent working memory training studies indicated that children's numerical abilities can be improved by working memory training (Kroesbergen, van't Noordende, & Kolkman, 2012; Kuhn & Holling, 2014). Nevertheless, all of these studies did not specifically investigate children or adults with DD. Hence, it remains an open question whether visual-spatial working memory training might be effective in DD remediation. Further research is needed to investigate whether sustained gains in numerical and/or arithmetic performance of individuals with DD can be achieved by employing visual-spatial working memory training.

5. Conclusion

In the present study, we investigated whether previously observed poorer performance of individuals with DD in the number line estimation task may be related to a general deficit in mapping numbers onto space. We found that adults with DD performed worse when having to learn any new non-linear number-to-space mapping. Additionally, we observed that those with DD seemed to try to compensate for their impaired spatial-numerical competencies by relying on reference points, especially in the linear layout condition. Therefore, we conclude that adults with DD seem to present with a general impairment of mapping symbolic numbers onto space, which is not limited to one particular (e.g., linear) number-space mapping. This finding can be explained by a deficit in visual-spatial memory, which has been previously found to be impaired in DD. Importantly, the linear layout usually used in number line estimation tasks seems not well suited to study deficits of number-to-space mappings in dyscalculia because at least adults with DD can partially compensate for the observed deficits by using reference points. Therefore, the severity of their deficits of number-to-space mapping deficit might be underestimated when using the standard linear number line estimation task.

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Appendix A

Table A1

Table A1
Functions used in the fitting process.

Function	Model	Coefficients
Linear function	$f(x) = a * x + b$	a, b
Exponential function	$f(x) = a * e^{b*x} + c$	a, b, c
Logarithmic function	$f(x) = a * \ln(x + b) + c$	a, b, c
Sigmoid function	$f(x) = \frac{a}{1+e^{-b*x+c}}$	a, b, c
Inverse sigmoid function	$f(x) = a*\ln(\frac{b*x}{1-b*x}) + c$	a, b, c

References

Andersson, U., & Lyxell, B. (2007). Working memory deficit in children with mathematical difficulties: A general or specific deficit? *Journal of Experimental Child Psychology, 96*(3), 197–228. http://dx.doi.org/10.1016/j.jecp.2006.10.001

Ashkenazi, S., Rubinsten, O., & Henik, A. (2009). Attention, automaticity, and developmental dyscalculia. *Neuropsychology, 23*(4), 535–540. http://dx.doi.org/10.1037/a0015347

Barr, D. J., Levy, R., Scheepers, C., & Tily, H. J. (2013). Random effects structure for confirmatory hypothesis testing: Keep it maximal. *Journal of Memory and Language, 68*(3), 255–278.

Bartelet, D., Ansari, D., Vaessen, A., & Blomert, L. (2014). Cognitive subtypes of mathematics learning difficulties in primary education. *Research in Developmental Disabilities, 35*(3), 657–670. http://dx.doi.org/10.1016/j.ridd.2013.12.010

Barth, H. C., & Paladino, A. M. (2011). The development of numerical estimation: Evidence against a representational shift. *Developmental Science, 14*(1), 125–135. http://dx.doi.org/10.1111/j.1467-7687.2010.00962.x

Bates, D., Maechler, M., Bolker, B., & Walker, S. (2014). *lme4: Linear mixed-effects models using Eigen and S4*. Retrieved from http://CRAN.R-project.org/package=lme4

Beddington, J., Cooper, C. L., Field, J., Goswami, U., Huppert, F. A., Jenkins, R., et al. (2008). The mental wealth of nations. *Nature, 455*(7216), 1057–1060. http://dx.doi.org/10.1038/4551057a

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry, and Allied Disciplines, 46*(1), 3–18. http://dx.doi.org/10.1111/j.1469-7610.2004.00374.x

De Smedt, B., Noël, M.-P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education, 2*(2), 48–55. http://dx.doi.org/10.1016/j.tine.2013.06.001

Defever, E., Göbel, S. M., Ghesquiere, P., & Reynvoet, B. (2014). Automatic number priming effects in adults with and without mathematical learning disabilities. *Frontiers in Psychology, 5*, 4. http://dx.doi.org/10.3389/fpsyg.2014.00004

Dumontheil, I., & Klingberg, T. (2012). Brain activity during a visuospatial working memory task predicts arithmetical performance 2 years later. *Cerebral Cortex, 22*(5), 1078–1085.

Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links between the intuitive sense of number and formal mathematics ability. *Child Development Perspectives, 7*(2), 74–79. http://dx.doi.org/10.1111/cdep.12019

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*(1), 4–15. http://dx.doi.org/10.1177/00222194040370010201

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology, 47*(6), 1539–1552. http://dx.doi.org/10.1037/a0025510

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & Catherine DeSoto, M. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology, 88*(2), 121–151. http://dx.doi.org/10.1016/j.jecp.2004.03.002

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development, 78*(4), 1343–1359. http://dx.doi.org/10.1111/j.1467-8624.2007.01069.x

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology, 33*(3), 277–299. http://dx.doi.org/10.1080/87565640801982361

Goswami, U. (2008). *Learning difficulties: Future challenges*. London: The Government Office for Science.

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. *Developmental Psychology, 48*(5), 1229–1241. http://dx.doi.org/10.1037/a0027433

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology, 93*(3), 615–626. http://dx.doi.org/10.1037/0022-0663.93.3.615

Hollands, J. G., & Dyre, B. P. (2000). Bias in proportion judgments: The cyclical power model. *Psychological Review, 107*(3), 500–524.

Huber, S., Moeller, K., & Nuerk, H.-C. (2014). Dissociating number line estimations from underlying numerical representations. *Quarterly Journal of Experimental Psychology, 67*(5), 991–1003. http://dx.doi.org/10.1080/17470218.2013.838974

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74*(3), 834–850.

Karolis, V., Iuculano, T., & Butterworth, B. (2011). Mapping numerical magnitudes along the right lines: Differentiating between scale and bias. *Journal of Experimental Psychology: General, 140*(4), 693–706. http://dx.doi.org/10.1037/a0024255

Kaufmann, L., Mazzocco, M. M., Dowker, A., von Aster, M., Goebel, S., Grabner, R., et al. (2013). Dyscalculia from a developmental and differential perspective. *Frontiers in Psychology, 4*, 516. http://dx.doi.org/10.3389/fpsyg.2013.00516

Kaufmann, L., & von Aster, M. (2012). The diagnosis and management of dyscalculia. *Deutsches Ärzteblatt International, 109*(45), 767–777. http://dx.doi.org/10.3238/arztebl.2012.0767

Kennet-Cohen, T., Bronner, S.B.-S., & Intrator, N. (2008). *Different approaches for combining scores on a test battery for the diagnosis of learning disabilities*. Jerusalem: National Institute for Testing and Evaluation.

Kroesbergen, E. H., van't Noordende, J. E., & Kolkman, M. E. (2012). Training working memory in kindergarten children: Effects on working memory and early numeracy. *Child Neuropsychology, 20*(1), 23–37. http://dx.doi.org/10.1080/09297049.2012.736483

Kuhn, J.-T., & Holling, H. (2014). Number sense or working memory? The effect of two computer-based trainings on mathematical skills in elementary school. *Advances in Cognitive Psychology, 10*(2), 59–67.

Kuznetsova, A., Brockhoff, P. B., & Christensen, R. H. (2014). *lmerTest: Tests for random and fixed effects for linear mixed effect models (lmer objects of lme4 package) R package version 2.0-6*. Retrieved from http://CRAN.R-project.org/package=lmerTest

Landerl, K. (2013). Development of numerical processing in children with typical and dyscalculic arithmetic skills – A longitudinal study. *Frontiers in Psychology, 4*, 459. http://dx.doi.org/10.3389/fpsyg.2013.00459

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9-year-old students. *Cognition, 93*(2), 99–125. http://dx.doi.org/10.1016/j.cognition.2003.11.004

Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: Two learning disorders with different cognitive profiles. *Journal of Experimental Child Psychology, 103*(3), 309–324. http://dx.doi.org/10.1016/j.jecp.2009.03.006

Landerl, K., & Kolle, C. (2009). Typical and atypical development of basic numerical skills in elementary school. *Journal of Experimental Child Psychology, 103*(4), 546–565. http://dx.doi.org/10.1016/j.jecp.2008.12.006

Langsrud, Ø. (2003). ANOVA for unbalanced data: Use Type II instead of Type III sums of squares. *Statistics and Computing, 13*(2), 163–167. http://dx.doi.org/10.1023/A:1023260610025

LeFevre, J.-A., Jimenez Lira, C., Sowinski, C., Cankaya, O., Kamawar, D., & Skwarchuk, S.-L. (2013). Charting the role of the number line in mathematical development. *Frontiers in Psychology, 4*, 641. http://dx.doi.org/10.3389/fpsyg.2013.00641

Mejias, S., Grégoire, J., & Noël, M.-P. (2012). Numerical estimation in adults with and without developmental dyscalculia. *Learning and Individual Differences, 22*(1), 164–170. http://dx.doi.org/10.1016/j.lindif.2011.09.013

Melby-Lervåg, M., & Hulme, C. (2013). Is working memory training effective? A meta-analytic review. *Developmental Psychology, 49*(2), 270–291. http://dx.doi.org/10.1037/a0028228

Metcalfe, A. W. S., Ashkenazi, S., Rosenberg-Lee, M., & Menon, V. (2013). Fractionating the neural correlates of individual working memory components underlying arithmetic problem solving skills in children. *Developmental Cognitive Neuroscience, 6*, 162–175. http://dx.doi.org/10.1016/j.dcn.2013.10.001

Moeller, K., & Nuerk, H.-C. (2011). Psychophysics of numerical representation. *Zeitschrift für Psychologie, 219*(1), 64–70. http://dx.doi.org/10.1027/2151-2604/a000048

Mussolin, C., Martin, R., & Schiltz, C. (2011). Relationships between number and space processing in adults with and without dyscalculia. *Acta Psychologica, 138*(1), 193–203. http://dx.doi.org/10.1016/j.actpsy.2011.06.004

Mussolin, C., & Noel, M.-P. (2008). Specific retrieval deficit from long-term memory in children with poor arithmetic facts abilities. *Open Psychology Journal, 1*, 26–34.

Parsons, S., & Bynner, J. (2005). *Does numeracy matter more?*.

R Development Core Team (2015). *R: A language and environment for statistical computing*. Vienna, Austria: R Foundation for Statistical Computing Retrieved from http://www.R-project.org/

Rousselle, L., & Noël, M.-P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*(3), 361–395. http://dx.doi.org/10.1016/j.cognition.2006.01.005

Rousselle, L., & Noel, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*(3), 361–395. http://dx.doi.org/10.1016/j.cognition.2006.01.005

Rubinsten, O., & Henik, A. (2005). Automatic activation of internal magnitudes: A study of developmental dyscalculia. *Neuropsychology, 19*(5), 641–648. http://dx.doi.org/10.1037/0894-4105.19.5.641

Shalev, R. S., Auerbach, J., Manor, O., & Gross-Tsur, V. (2000). Developmental dyscalculia: Prevalence and prognosis. *European Child & Adolescent Psychiatry, 9*, II58–II64.

Shalev, R. S., Manor, O., & Gross-Tsur, V. (2005). Developmental dyscalculia: A prospective six-year follow-up. *Developmental Medicine and Child Neurology, 47*(2), 121–125.

Siegler, R. S., & Booth, J. L. (2005). *Development of numerical estimation: A review*. Boca Raton, FL: CRC Press.

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science, 14*(3), 237–250. http://dx.doi.org/10.1111/1467-9280.02438

Slusser, E. B., Santiago, R. T., & Barth, H. C. (2013). Developmental change in numerical estimation. *Journal of Experimental Psychology: General, 142*(1), 193–208. http://dx.doi.org/10.1037/a0028560

Szucs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2013). Developmental dyscalculia is related to visuo-spatial memory and inhibition impairment. *Cortex, 49*(10), 2674–2688. http://dx.doi.org/10.1016/j.cortex.2013.06.007

Szűcs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2014). Cognitive components of a mathematical processing network in 9-year-old children. *Developmental Science, 17*(4), 506–524. http://dx.doi.org/10.1111/desc.12144

Wilson, A. J., & Dehaene, S. (2007). *Number sense and developmental dyscalculia*. New York: Guildford Press.